# Type-3 2-D Multimode IIR Filter Architecture and the Corresponding Symmetry Filter's Error Analysis


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*Abstract*—**In this paper, we propose a low-cost high-speed Type-3 two-dimensional (2-D) multimode symmetry IIR filter architecture including four different operation modes: diagonal symmetry mode (DSM), fourfold rotational symmetry mode (FRSM), quadrantal symmetry mode (QSM), and octagonal symmetry mode (OSM). For multimode purpose, the Type-3 transfer function is applied and the interconnections to achieve corresponding four symmetry filter architectures are proposed. The proposed 2-D IIR filter structure has lower number of adders and shorter critical path than the published Type-1 IIR filter structure without sacrificing multipliers. Furthermore, the theoretical error analysis of 2-D IIR filter architectures with diagonal symmetry and four-fold rotational symmetry is studied.**


## I. INTRODUCTION

Nowadays, two-dimensional (2-D) digital filters are widely employed in modern and commercial digital signal processing (DSP) systems such as image enhancement [1], frequency response analysis [2-6], and beamformer [7, 8]. Even though 2-D digital filters can be performed and practiced in a general-purpose processor, the real time signal or information processing and integration to commercial product are probably difficult. To shrink the bottleneck, an application-specific integrated circuit (ASIC) approach is applied in 2-D IIR filter implementation. Using an ASIC approach, the throughput rate can be sped up and cost could be reduced. Several 2-D VLSI filter architectures have been briefly reviewed and studied in [9], and an existing ASIC approach has been applied to the design of beam filter [7, 8] and image processing [8], and 2-D symmetry filters [10-15]. The significant feature of filters studied in [10-15] is that they jointly exhibit symmetry in the magnitude function of the frequency response. This filter symmetry property can be used to reduce the number of multipliers for the implementation. However, how to obtain a Type-3 cost-effective and high-speed 2-D multimode symmetry filter including four symmetries has not yet explored. The proposed multimode filter has lower number of adders without sacrificing multipliers and higher speed compared with that in [12]. On the other hand, it will be a challenge to determine the bit width of the filter before the ASIC implementation. If the larger bit width is applied, the cost will be larger. On the contrast, if the lower bit width is selected, the error will be large or unstable. Although error analysis for Type-3 quadrantal- and octagonal-symmetry filter have been issued in [15], the error analysis for diagonal and four-fold rotational symmetry filters has not been yet discussed. The paper is organized as follows. Section II briefly reviews the separable denominator filter transfer function. In Section III, 2-D multimode symmetry filter architecture and the corresponding interconnection to configure four symmetry modes are proposed. Next, error analysis expression for diagonal and four-fold rotational symmetry filter is discussed in Section IV. In Section V, the comparison of Type-3 and Type-1 multimode symmetry filter structures is discussed. Finally, the conclusion is remarked in Section VI.

## II. UNDERSTANDING OF SEPARABLE DENOMINATOR FILTER

The generalized transfer function of a 2-D IIR digital filter can be represented as [12]

$$H(z_1,z_2)=\frac{Y(z_1,z_2)}{X(z_1,z_2)}=\frac{\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}a_{ij}z_1^{-i}z_2^{-j}}{\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}b_{ij}z_1^{-i}z_2^{-j}} \tag{1}$$

where $X(z_1, z_2)$ and $Y(z_1, z_2)$ denote the input and output of the filter, respectively, $a_{ij}$ and $b_{ij}$ denote the numerator and denominator coefficients, respectively, and $b_{00}=1$, $N_1 \times N_2$ is the order of the IIR filter. Throughout this paper, assume that $N_1=N_2=N$ for the filter order and $M$ size input image is fed to the following structures in raster-scan mode. Thus, the delay $z_2^{-1}=z^{-1}$ and $z_1^{-1}=z^{-M}$, where $z^{-1}$ and $M$ denote a unit delay element and the width of input image, respectively. Considering the separable denominator transfer function, (1)

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can be described in (2) [12]:

$$H(z_1,z_2)=\frac{Y(z_1,z_2)}{X(z_1,z_2)}=\frac{\sum_{i=0}^{N}\sum_{j=0}^{N}a_{ij}z_1^{-i}z_2^{-j}}{\left(1-\sum_{i=1}^{N}b_{i0}z_1^{-i}\right)\left(1-\sum_{j=1}^{N}b_{0j}z_2^{-j}\right)} \quad (2)$$

The Type-3 separable denominator transfer function of the 2-D filter can be recast in (3)[14].

$$H(z_1,z_2)=\frac{Y(z_1,z_2)}{Y_3(z_1,z_2)}\cdot\frac{Y_3(z_1,z_2)}{X(z_1,z_2)} \quad (3)$$

where $Y(z_1, z_2) / Y_3(z_1, z_2)$ is defined in (4).

$$Y=Y_3+\sum_{j=1}^{N}b_{0j}z_2^{-j}Y \quad (4)$$

$Y=Y(z_1, z_2)$ and $Y_3=Y_3(z_1, z_2)$. Therefore, $Y_3(z_1, z_2) / X(z_1, z_2)$ can be generally represented as follows.

$$Y_3=\sum_{i=0}^{N}\sum_{j=0}^{N}a_{ij}z_1^{-i}z_2^{-j}X+\sum_{i=1}^{N}b_{i0}z_1^{-i}Y_3 \quad (5)$$

where $X=X(z_1, z_2)$. Since the image is raster scanned, the delay $z_1^{-1}=z^{-M}$ is realized by a shift-register (SR) with size of $M$.

## III. Low-Cost 2-D Multimode IIR Filter Architecture with Four Symmetries

In [12, 13], we have demonstrated that symmetry in the frequency response can result in fewer multipliers in the 2-D filter implementation. Four Type-3 2-D IIR filter architectures with diagonal, four-fold rotational, quadrantal, and octagonal symmetries are proposed in [13, 15]. In order to support multiple symmetry functions, one 2-D multimode IIR filter architecture with four symmetry modes: diagonal symmetry mode (DSM), fourfold rotational symmetry mode (FRSM), quadrantal symmetry mode (QSM), and octagonal symmetry mode (OSM) is proposed in Fig. 1. The 2-D multimode symmetry filter architecture as shown in Fig. 1 can be devised according to the following three observations. First, it is observed that signal paths are dispatched after the independent $a_{ij}$ coefficient multiplier for the Type-3 symmetry filters in [13, 15]. According to the delay arrangement analysis discussed in [12], the Type-3 diagonal, fourfold rotational, quadrantal, and octagonal symmetry filters have regular delay arrangement. Thus, the four Type-3 symmetry structures are capable of being the basis for the four configuration modes for the Type-3 2-D multimode IIR filter. Second, it can be observed in (3) that the Type-3 symmetry filter consists of two transfer functions: $Y(z_1, z_2) / Y_3(z_1, z_2)$ and $Y_3(z_1, z_2) / X(z_1, z_2)$, with the transfer function being the same for the four individual symmetry filters as shown in [13, 15]. The block diagram of $Y(z_1, z_2) / Y_3(z_1, z_2)$ for $N$=3 is depicted as Block 1 on the right hand side in Fig. 1. Block1 requires 3 multipliers and 3 adders. Next, we consider the transfer function which is different for each of the four individual symmetry filters. To construct Block 2 of the multimode filter, we need 3 multipliers for the denominator $b_{0j}$, and 11 multipliers for the numerator. It is noted that the requirements on the numerator multipliers are similar to descriptions in [12], thus we do not repeat it here. Third, in [13, 15], the interconnection control is only needed for the four transfer functions. The multiplication connections and internal connections are controlled by interconnection boxes (IBs) to accomplish the four-mode operations, where IB performs either connection or disconnection task for each signal path. According to the connections of the four individual symmetry filter structures, 11 IBs are needed for the internal connections in Block 2. Note that the solid line feeding through the IB means that this signal path is not controlled by four modes. As a consequence, considering the three observations mentioned above, the 2-D multimode IIR filter with four symmetry modes can be obtained in Fig. 1, and the interconnections difference among four configurations of the proposed multimode filter architecture is highlighted in Fig. 2.

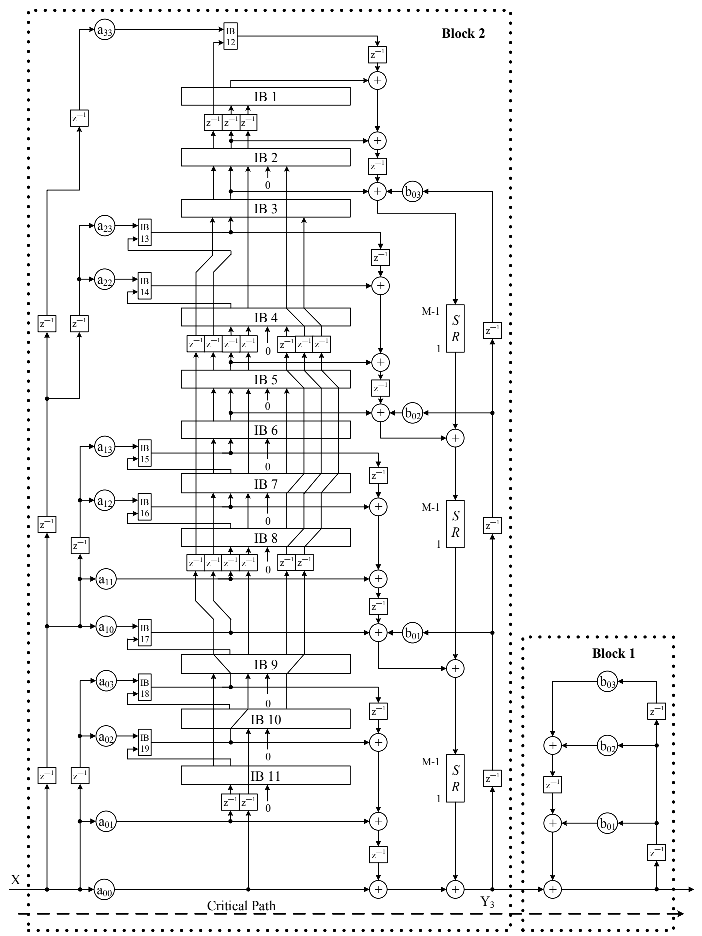


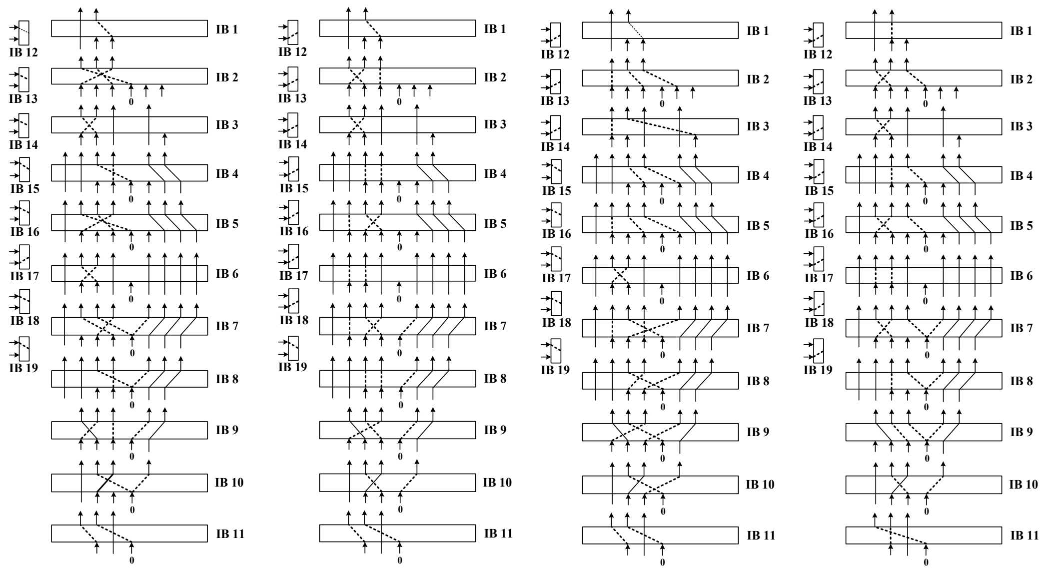

Fig. 1. Proposed 2-D multimode IIR filter architecture with four symmetries for $N$=3.

## IV. Error Analysis for Type-3 and Type-1 2-D Diagonal-Symmertry IIR Filter Architecture

In this section, applying the constraints of $a_{ij}=a_{ji}$ and $b_{k0}=b_{0k}$ for all $i, j, k$. to (2), the diagonal symmetry filter equations can be obtained as (6) and (7), respectively [14].

$$Y=Y_3+\sum_{j=1}^{N}b_{0j}z_2^{-j}Y \quad (6)$$

$$Y_3=\sum_{j=1}^{N}b_{0j}z_1^{-j}Y_3+\sum_{i=0}^{N}a_{ii}z_1^{-i}z_2^{-i}X+\sum_{i=0}^{N-1}\sum_{j=i+1}^{N}a_{ij}(z_1^{-i}z_2^{-j}+z_1^{-j}z_2^{-i})X \quad (7)$$




(a) (b) (c) (d)

Fig. 2. Interconnections of (a) DSM, (b) FRSM, (c) QSM, (d) OSM for proposed 2-D multimode IIR filter architecture.

According to (6) and (7) with $N$=3, the Type-3 2-D diagonal-symmetry IIR filter structure with separable denominator is repetitively depicted in Fig 3. Due to the product in fixed-point implementation, the quantization errors/roundoff noises propagating through the filter have been addressed in [3, 4, 15]. Using the same assumption and approach as mentioned in [3, 15], the noise sources are uncorrelated, wide-sense stationary process, and uniform distribution in linear noise model and linear decomposition can be applied. According to the method and descriptions in [3, 15], the error signal notations are indicated in Fig. 3, where the signal notations $e_1$ and $e_2$ denote the linear errors as (8) and (9), respectively.

$$e_1 = \sum_{j=1}^{N} e_{b0j} \tag{8}$$

$$e_2 = \sum_{j=1}^{N} e_{b0j} + \sum_{i=0}^{N} e_{aii} + 2 \cdot \sum_{i=0}^{N-1} \sum_{j=i+1}^{N} e_{aij} \tag{9}$$

The total variance of quantization error is derived as

$$\sigma^2_{\text{Type3\_Dia}} = N\sigma_e^2 \sum_{n=-\infty}^{\infty} \left|h_{b2}[n]\right|^2 + \left[N + (N+1)^2\right]\sigma_e^2 \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left|h_{b12}[m,n]\right|^2 \tag{10}$$

, where $\sigma_e^2 = 2^{-2B}/12$ , B is the fractional bit width after quantization, and $h_{b2}[n]$ and $h_{b12}[m,n]$ are defined in (11) and (12), respectively.

$$h_{b2}[n] \overset{Z}{\longleftrightarrow} 1 \Big/ \left(1 - \sum_{j=0}^{N} b_{0j} z_2^{-j}\right) \tag{11}$$

$$h_{b12}[m,n] \overset{Z}{\longleftrightarrow} 1 \Big/ \left(\left(1 - \sum_{i=0}^{N} b_{0i} z_1^{-i}\right) \cdot \left(1 - \sum_{j=0}^{N} b_{0j} z_2^{-j}\right)\right). \tag{12}$$

It is worthy of noting that the formulation of the total variance of quantization error of the Type-3 diagonal symmetry filter architecture is the same as that of Type-3 quadrantal and octagonal symmetry filter structure in [15]. In the similar way, we can derive the variance of quantization error of Type-3 four-fold rotational symmetry filter. For completeness, the error analysis of Type-1 diagonal and four-fold rotational symmetry filters are listed as follows.

$$\sigma^2_{\text{Type1\_D}ia} = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left|h_{b12}[m,n]\right|^2 + \left(2N + 1 + \frac{N(N+1)}{2}\right)\sigma_e^2 \sum_{n=-\infty}^{\infty} \left|h_{b2}[n]\right|^2 \tag{13}$$

$$\sigma^2_{\text{Type1\_FF}} = N\sigma_e^2 \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \left|h_{b12}[m,n]\right|^2 + \left(N + v + (u - v + 1)(N - u + v)\right)\sigma_e^2 \sum_{n=-\infty}^{\infty} \left|h_{b2}[n]\right|^2 \tag{14}$$

## V. Comparison

In this section, the comparison results of filter architectures are tabulated in Table 1 for $N$=3. The performance metrics are number of adders with two inputs, number of multipliers and critical path, where $T_m$ and $T_a$ denote the operation time required by one multiplier and one adder, respectively. From Table 1, compared with Type 1 2-D multimode IIR filter architecture, it can be seen that the proposed one (i.e., Type 3 multimode IIR filter) is capable of providing lower number of adders and shorter critical path without sacrificing number of multipliers. The proposed 2-D multimode symmetry filter structure can result in the number of adders saving by 27.39% and lower critical path (i.e., $T_m$+2$T_a$) while comparing with the Type-1 2-D multimode symmetry filter in [12]. Compared with four Type-3 individual symmetry filters [13, 15] in Table 1, the proposed multimode IIR filter attains the number of multipliers saving by 65.31%. Thus, the proposed 2-D multimode IIR filter architecture can achieve lower cost and higher speed from the viewpoint of the architecture level.

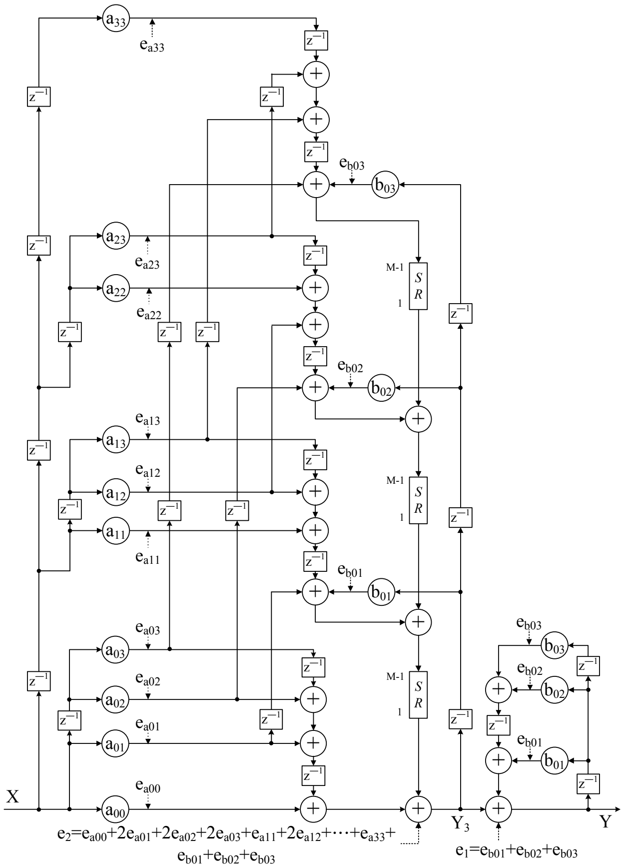



Fig. 3. Type-3 2-D diagonal-symmetry IIR filter architecture for $N$=3 [13].

ACKNOWLEDGEMENT

This work was supported in part by the Ministry of Science and Technology (MOST) under Grant MOST 106-2221-E-009-028-MY3 and MOST 103-2221-E-009-099-MY3.

## VI. CONCLUSION

One Type-3 low-cost and high-speed 2-D multimode symmetry IIR filter architecture including four different configurations: DSM, FRSM, QSM, and OSM is proposed. The proposed filter structure has lower number of adders and shorter critical path than that the Type-1 filter structure without sacrificing multipliers. Furthermore, the error analysis of the 2-D IIR filter structure with diagonal symmetry is provided.

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TABLE 1 COMPARISON RESULTS AMONG IIR FILTER ARCHITECTURES FOR $N$=3.

| 2-D Filter Architecture | # of adders with two inputs | | # of multipliers | | Critical Path (with tree method) | Multimode (Support four symmerty configurations) |
|---|---|---|---|---|---|---|
| Type-1 multimode symmetry filter [12] | 29 | 100% | 17 | 100% | $T_m$+3$T_a$ | Yes |
| Type-3 diagonal symmerty filter [13] | 21 | 72.41% | 16 | 94.12% | $T_m$+2$T_a$ | No |
| Type-3 four-fold rotational symmerty filter [13] | 21 | 72.41% | 10 | 58.82% | $T_m$+2$T_a$ | No |
| Type-3 quadrantal symmerty filter [15] | 21 | 72.41% | 14 | 82.35% | $T_m$+2$T_a$ | No |
| Type-3 octagonal symmerty filter [15] | 21 | 72.41% | 9 | 52.94% | $T_m$+2$T_a$ | No |
| Proposed Type-3 multimode symmetry filter | 21 | 72.41% | 17 | 100% | $T_m$+2$T_a$ | Yes |