New 2-D Quadrantal- and Diagonal-Symmetry Filter Architectures Using Delta Operator

Pei-Yu Chen and Lan-Da Van
Department of Computer Science,
National Chiao Tung University, Hsinchu, Taiwan
e-mail: pychen@viplab.cs.nctu.edu.tw ; ldvan@cs.nctu.edu.tw

I. H. Khoo and Hari C. Reddy, *IEEE Fellow*
Department of Elecrical Engineering, California State University, Long Beach, USA
e-mail: I-Hung.Khoo@csulb.edu ; hreddy@csulb.edu

Abstract—**In this paper, we propose two new two-dimensional (2-D) IIR filter architectures with quadrantal and diagonal magnitude symmetry using delta operator. These filter architectures can attain lower coefficient sensitivity in comparison with the traditional shift-operator formulation (i.e., the conventional *z*-domain filters). The IIR filter structures presented not only achieve lower coefficient sensitivity without sacrificing the number of multipliers, but in fact reducing the number of multipliers by 36.36% compared with the previous corresponding quadrantal symmetry filter structures using delta operator with order *N*=3.**

I. Introduction

Two-dimensional (2-D) digital filters have been extensively employed in many research and practical digital signal processing (DSP) applications [1, 2, 3]. In the past decade, the symmetry properties applied to digital filters are widely discussed in [4-11]. From these studies, the number of multipliers can be potentially decreased if the proper symmetry is adopted. Concerning the hardware implementation, high speed 2-D digital filters [12] and the related symmetry digital filter architectures [13-17] have been devised in an application-specific integrated circuit (ASIC) approach. The design of quadrantal symmetry filter structure using delta operator, where the structure combined the advantages of delta operator and quadrantal symmetry was given in [15]. It is known that delta operator based digital filter can offer better numerical accuracy and lower coefficient sensitivity in narrow-band filter designs compared with that in traditional *z*-domain, and further the quadrantal symmetry can be applied to reduce the number of multipliers in a digital filter. However, how to improve the quadrantal symmetry filter architecture [15] and devise other symmetry filter architectures have not yet been explored. Thus, it is our motivation to propose an improved quadrantal symmetry filter architecture and a new diagonal symmetry filter architecture with lower coefficient sensitivity in an ASIC approach. In this paper, the preliminaries of 2-D filter transfer functions and symmetry using delta operator are briefly introduced in Sections II. In Section III and IV, 2-D quadrantal- and diagonal-symmetry filter architecture using delta operator with separable denominator transfer function are presented. Next, the comparison and discussion is given in Section V. Finally, the conclusion is marked in Section VI.

II. Preliminaries

The general 2-D filter transfer function using delta operator in *r* domain is released in [15] as follows.

$$H(\gamma_1,\gamma_2)=\frac{Y(\gamma_1,\gamma_2)}{X(\gamma_1,\gamma_2)}=\frac{\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}c_{ij}\gamma_1^{-i}\gamma_2^{-j}}{1-\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}d_{ij}\gamma_1^{-i}\gamma_2^{-j}} \quad (1)$$

, where the relationship between transfer function $H(z_1, z_2)$ in the *z*-domain and transfer function $H(\gamma_1, \gamma_2)$ in the γ-domain is

$$H(\gamma_1,\gamma_2)=H(z_1,z_2)\big|_{z_i=(1+T_i\gamma_i)\ ,i=1,2} \quad (2)$$

Adopting the symmetry features, the transfer function with separable denominator can be expressed in (3).

$$H(\gamma_1,\gamma_2)=\frac{Y(\gamma_1,\gamma_2)}{X(\gamma_1,\gamma_2)}=\frac{\sum_{i=0}^{N_1}\sum_{j=0}^{N_2}c_{ij}\gamma_1^{-i}\gamma_2^{-j}}{\left(1-\sum_{i=1}^{N_1}d_{i0}\gamma_1^{-i}\right)\left(1-\sum_{i=1}^{N_2}d_{0j}\gamma_2^{-j}\right)} \quad (3)$$

The separable denominator transfer function has following advantages. First, it is easy to check and maintain the stability of the two 1-D polynomials by solving the poles. Second, fewer hardware multipliers are needed in realization. Without loss of generality, the filter order is N_1=N_2=N and an image size is MxM throughout this paper.

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III. 2-D Quadrantal Symmetry Filter Architecture Using Delta Operator

In this section, the quadrantal symmetry filter with delta operator will be discussed. The property of quadrantal property mentioned in [4, 6-11] is that the magnitude squared function is

$$F(\theta_1,\theta_2)=F(-\theta_1,\theta_2)=F(\theta_1,-\theta_2)=F(-\theta_1,-\theta_2),\ \forall(\theta_1,\theta_2) \quad (4)$$

,where $F(\theta_1,\theta_2)=\mathrm{P}(\gamma_1,\gamma_2)\cdot \mathrm{P}\left(\frac{-\gamma_1}{1+T\gamma_1},\frac{-\gamma_2}{1+T\gamma_2}\right)\Bigg|_{\gamma_i=\frac{e^{j\theta_i}-1}{T},\ i=1,2}$.

where T denotes the sampling period. The transfer function of the quadrantal symmetry filter using delta operator is repeated in (5) [15].

$$H(\gamma_1,\gamma_2)=\frac{Y(\gamma_1,\gamma_2)}{X(\gamma_1,\gamma_2)}=\frac{\sum_{i=0}^{N}\sum_{j=0,2,4\ldots}^{N}c_{ij}\gamma_1^{-i}(\gamma_2^{-2}+\gamma_2^{-1})^{j/2}}{\left(1-\sum_{i=1}^{N_1}d_{i0}\gamma_1^{-i}\right)\left(1-\sum_{i=1}^{N_2}d_{0j}\gamma_2^{-j}\right)} \quad (5)$$

The quadrantal symmetry filter structure using delta operator has been released in [15]. However, the numbers of multipliers and adders have not yet been further reduced. Using the coefficient matrix distribution of the quadrantal symmetry property for N=3 as shown in Fig. 1, the numerator coefficients in square box can be shared.

$$\begin{array}{c} \\ \gamma_2^{-0} \\ \gamma_2^{-1} \\ \gamma_2^{-2} \\ \gamma_2^{-3}\end{array}\begin{array}{c}\begin{array}{cccc}\gamma_1^{-0} & \gamma_1^{-1} & \gamma_1^{-2} & \gamma_1^{-3}\end{array}\\ \begin{bmatrix} c_{00} & c_{10} & c_{20} & c_{30} \\ \boxed{c_{02}} & \boxed{c_{12}} & \boxed{c_{22}} & \boxed{c_{32}} \\ c_{02} & c_{12} & c_{22} & c_{32} \\ 0 & 0 & 0 & 0 \end{bmatrix}\end{array}$$

Fig. 1. Coefficient matrix distribution of the quadrantal symmetry.

Corresponding to (5) and Fig. 1, the block diagram of an improved quatrantal symmetry filter architecture using delta operator is shown in Fig. 2. In Fig. 2, the black box denotes one delay element z^{-1}, and the stripe box is the shift register, which realizes $z_1^{-1}=z^{-M}$ with size of M [14]. It is noted that the proposed separable denominator filter structure is different from that proposed in [15], where the shifted delay location in Fig. 2 using the register splitting method [12] can attain the multiplier sharing for diagonal symmetry filter architecture. The 2-D FIR filter architecture can be simplified in Fig. 3.

IV. 2-D Diagonal Symmetry Dilter Achitecture Using Delta Operator

The 2-D diagonal symmetry filter structure using delta operator will be more complicated if the regular modules [15] are used. According to the property mentioned in [8], the polynomial in r domain is $P(r_1, r_2)=P(r_2, r_1)$. The transfer function of the diagonal symmetry filter in r domain is

$$H(\gamma_1,\gamma_2)=\frac{\sum_{i=0}^{N}c_{ii}\gamma_1^{-i}\gamma_2^{-i}+\sum_{i=0}^{N-1}\sum_{j=i+1}^{N}c_{ij}(\gamma_1^{-i}\gamma_2^{-j}+\gamma_1^{-j}\gamma_2^{-i})}{\left(1-\sum_{i=1}^{N_1}d_{i0}\gamma_1^{-i}\right)\left(1-\sum_{i=1}^{N_2}d_{0j}\gamma_2^{-j}\right)} \quad (6)$$

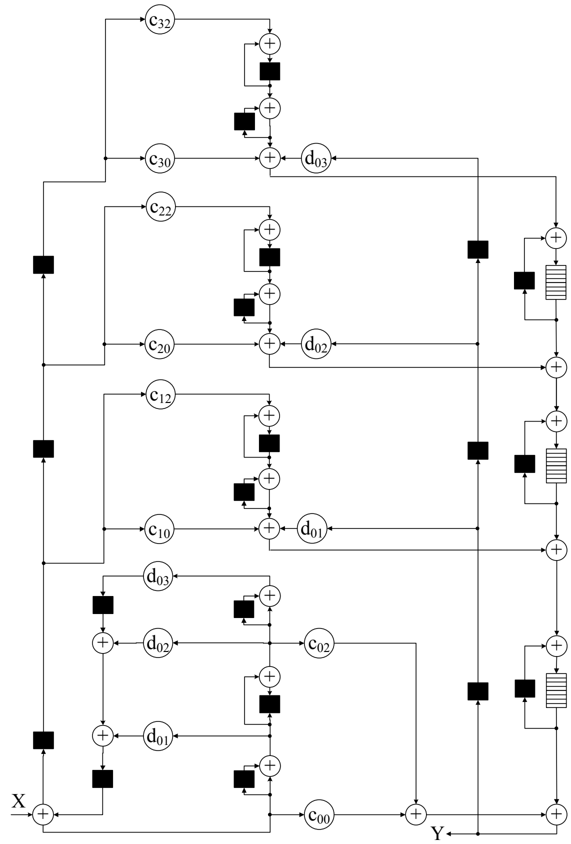

Fig. 2. 2-D IIR filter architecture with quadrantal symmetry using delta operator for N=3.

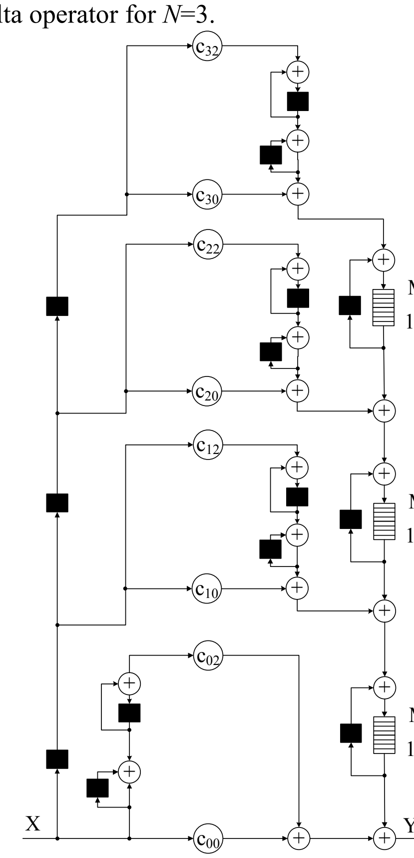

Fig. 3. 2-D FIR filter architecture with quadrantal symmetry using delta operator for N=3.

$$
\begin{array}{c} \\ \gamma_2^{-0} \\ \gamma_2^{-1} \\ \gamma_2^{-2} \\ \gamma_2^{-3} \end{array}
\begin{array}{c}
\begin{array}{cccc} \gamma_1^{-0} & \gamma_1^{-1} & \gamma_1^{-2} & \gamma_1^{-3} \end{array} \\
\begin{bmatrix}
c_{00} & c_{01} & c_{02} & c_{03} \\
\boxed{c_{01}} & c_{11} & c_{12} & c_{13} \\
\boxed{c_{02}} & \boxed{c_{12}} & c_{22} & c_{23} \\
\boxed{c_{03}} & \boxed{c_{13}} & \boxed{c_{23}} & c_{33}
\end{bmatrix}
\end{array}
$$

Fig. 4. Coefficient matrix distribution of the diagonal symmetry.

Using the coefficient matrix distribution of the diagonal symmetry property for N=3 as shown in Fig. 4, the numerator coefficients in square box can be shared. Thus, a new 2-D diagonal symmetry filter architecture using delta operator is shown in Fig. 5. The 2-D FIR filter architecture can be simplified in Fig. 6.

V. COMPARISON AND DISCUSSION

In this section, the architecture analysis and comparison results are shown in Table 1 in terms of the numbers of multipliers and adders in quantitative way, and the coefficient sensitivity in qualitative way. It is well-known that symmetry presented in the frequency responses of 2-D filters can be used to reduce the number of multipliers [14]. This work proposes one improved 2-D quadrantal-symmetry filter architecture and one new 2-D diagonal-symmetry filter architecture using delta operator for N=3. In similar way, the comparison of two new 2-D FIR filter architectures with quadrantal and diagonal symmetries using delta operator to other literatures are presented in Table 2 by setting coefficients in denominator as zero. In Table 1, compared with conventional quadrantal, and diagonal symmetry filter structures in [14], the proposed 2-D IIR filter structures using delta operator can achieve lower coefficient sensitivity without sacrificing the number of multipliers. In terms of number of multipliers, the proposed 2-D quadrantal symmetry filter structure using delta operator with N=3 can achieve the reduction by 36.36% compared with that in [15]. In terms of adders, it is known that the area of an adder is much less than that of a multiplier. In order to achieve fair comparison, the n-bit adder can be equivalently evaluated as $1/n$ nxn-bit multiplier using array multiplier approach [18]. According to the hardware implementation in [14], 10x10-bit multiplier and 10-bit full-adder with two inputs are reasonable assumptions to realize this evaluation. In Table 1 and Table 2, we can easily observe that the reduction of number of multipliers of the filter is still much larger than the increase of number of equivalent multipliers (i.e., the number of adders with two inputs).

VI. CONCLUSION

In this paper, two new symmetry filter architectures using delta operator are explored. Using these two new symmetry filter architectures, not only the fewer number of multipliers but also the lower coefficient sensitivity can be attained.

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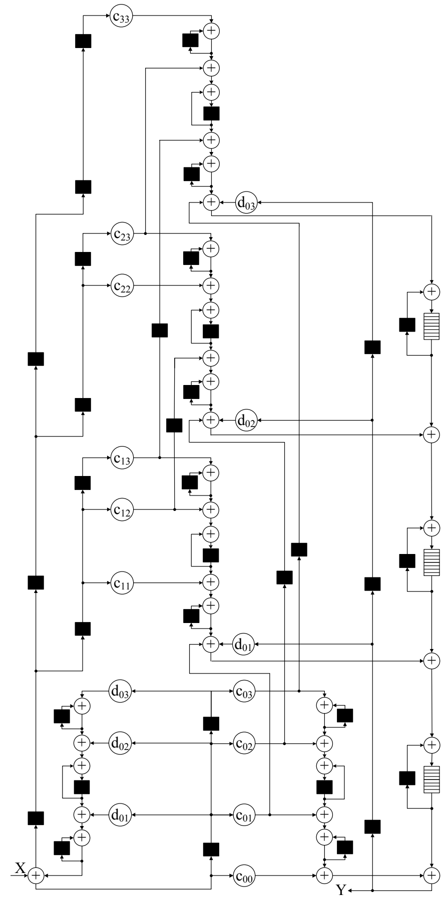

Fig. 5. 2-D IIR filter architecture with diagonal symmetry using delta operator for N=3.

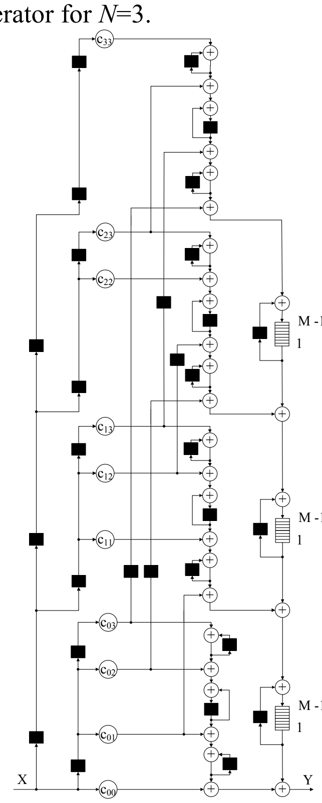

Fig. 6. 2-D FIR filter architecture with diagonal symmetry using delta operator for N=3.

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Table 1. Comparison of different 2-D IIR filters architectures with the order *N*.

Works	# of Multipliers	# of Multipliers for *N*=3		# of Adds with two inputs for *N*=3		Operator	Coefficient Sensitivity※3
		#	%	#	Equivalent # of 10x10-bit mul.		
[12]	$2(N+1)^2-1$	31 for Gen	100%	30	3.0	Shift	●
[14] Type-1	$\{\frac{1}{2}(N+1)^2+\frac{v}{2}(N+1)+2N,\ \frac{1}{2}(N+1)^2+\frac{5}{2}N+\frac{1}{2}\}$ ※1※2	{14, 16}	{45.16%, 51.61%}	{21, 21}	{2.1, 2.1}	Shift	●
[15]	$(N+1)^2+2N$	22 for Qua	70.97%	55	5.5	Delta	●●
[17]	$\frac{1}{2}(N+1)^2+\frac{v}{2}(N+1)+2N$ ※2	14 for Qua	45.16%	21	2.1	Shift	●
This work	$\left\{ \begin{cases} \frac{1}{2}(N+1)^2+2N, & \text{for } N \text{ is odd} \\ (N/2+1)(N+1)+2N, & \text{for } N \text{ is even} \end{cases},\ \frac{1}{2}(N+1)^2+\frac{5}{2}N+\frac{1}{2} \right\}$ ※1	{14, 16}	{45.16%, 51.61%}	{27, 39}	{2.7, 3.9}	Delta	●●

※1: {a, b} = {for quadrantal, for diagonal}. ※2: $v=(N+1) \bmod 2$. ※3: More block circles indicate better numerical accuracy.

Table 2. Comparison of different 2-D FIR filters architectures with the order *N*.

Works	# of Multipliers	# of Multipliers for *N*=3		# of Adds with two inputs for *N*=3		Operator	Coefficient Sensitivity※3
		#	%	#	Equivalent # of 10x10-bit mul.		
[12]	$(N+1)^2$	16 for Gen	100%	15	1.5	Shift	●
[14] Type-1	$\{\frac{1}{2}(N+1)^2+\frac{v}{2}(N+1),\ \frac{1}{2}(N+1)^2+\frac{1}{2}N+\frac{1}{2}\}$ ※1※2	{8, 10}	{50%, 62.5%}	{15, 15 }	{1.5, 1.5}	Shift	●
[15]	$(N+1)^2$	16 for Qua	100%	30	3.0	Delta	●●
[17]	$\frac{1}{2}(N+1)^2+\frac{v}{2}(N+1)$ ※2	8 for Qua	50%	15	1.5	Shift	●
This work	$\left\{ \begin{cases} \frac{1}{2}(N+1)^2, & \text{for } N \text{ is odd} \\ (N/2+1)(N+1), & \text{for } N \text{ is even} \end{cases},\ \frac{1}{2}(N+1)^2+\frac{1}{2}N+\frac{1}{2} \right\}$ ※1	{8, 10}	{50%, 62.5%}	{18, 30}	{1.8, 3.0}	Delta	●●

※1: {a, b} = {for quadrantal, for diagonal}. ※2: $v=(N+1) \bmod 2$. ※3: More block circles indicate better numerical accuracy.