

New 2-D Filter Architectures with Quadrantal Symmetry and Octagonal Symmetry and Their Error Analysis

Pei-Yu Chen and Lan-Da Van

Department of Computer Science,
National Chiao Tung University, Hsinchu, Taiwan
e-mail: pychenster@gmail.com ; ldvan@cs.nctu.edu.tw

Hari C. Reddy, *Fellow, IEEE*, I. H. Khoo
Department of Electrical Engineering, California State
University, Long Beach, USA
e-mail: hreddy@csulb.edu ; I-Hung.Khoo@csulb.edu

Abstract—In this paper, two new two-dimensional (2-D) IIR and FIR filter architectures possessing quadrantal and octagonal symmetries are proposed. Furthermore, the theoretical error analysis for the proposed filters is also presented. Utilizing the presented error analysis, user can decide the bit width of the filters with satisfactory error tolerance.

I. INTRODUCTION

In the past decades, two-dimensional (2-D) digital filters have been widely used in a variety of digital signal processing applications such as frequency response analysis [1-3], image restoration [4], image enhancement [5] and beamformer [6, 7]. In order to enhance the speed of performance, the existing application-specific integrated circuit (ASIC) approach [8-10] has been applied to designing 2-D filter architecture. By adapting an ASIC approach, the designs can achieve low cost and high throughput performance. Along with filter symmetry property, studies in [11-12] including 2-D diagonal symmetry filter [13, 15], four-fold rotational symmetry filter [14], quadrantal symmetry filter [15], octagonal symmetry filter and multimode symmetry filter [16] are proposed. Utilizing symmetry features presented in the magnitude function of the frequency response, the number of multipliers can be reduced in ASIC implementation. Although Type 1 and Type 2 filter has been addressed in [16]; however, the Type 3 filter with quadrantal and octagonal symmetry and the corresponding error analysis of filter structures have not yet been explored. The paper is organized as follows. Section II briefly reviews the separable denominator filter transfer function. The proposed Type-3 2-D quadrantal symmetry IIR and FIR filter architectures, and the study of corresponding error analysis are discussed in Section III. Next, in Section IV, two new Type-3 IIR and FIR filter structures with octagonal symmetries are presented along with the error analysis expression for these filters. Finally, the conclusion is

remarked in Section V.

II. CONCISE REVIEW OF SEPARABLE DENOMINATOR FILTER

The general transfer function of a 2-D IIR quarter-plane digital filter can be represented as follows [16]

$$H(z_1, z_2) = \frac{Y(z_1, z_2)}{X(z_1, z_2)} = \frac{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} a_{ij} z_1^{-i} z_2^{-j}}{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} b_{ij} z_1^{-i} z_2^{-j}} \quad (1)$$

where $X(z_1, z_2)$ and $Y(z_1, z_2)$ denote the input and output of the filter, respectively, a_{ij} and b_{ij} denote the numerator and denominator coefficients, respectively, and $b_{00}=1$, $N_1 \times N_2$ is the order of the IIR filter. Throughout this paper, M -size input image data is fed to the following structures in raster-scan mode, and thus the delay $z_2^{-1}=z^{-l}$ and $z_1^{-1}=z^{-M}$, where z^{-l} and M denote a unit delay element and the width of input image, respectively. Considering the separable denominator transfer function, (1) can be rewritten as (2) [16]:

$$H(z_1, z_2) = \frac{Y(z_1, z_2)}{X(z_1, z_2)} = \frac{\sum_{i=0}^{N_1} \sum_{j=0}^{N_2} a_{ij} z_1^{-i} z_2^{-j}}{\left(1 - \sum_{i=1}^{N_1} b_{i0} z_1^{-i}\right) \cdot \left(1 - \sum_{j=1}^{N_2} b_{0j} z_2^{-j}\right)} \quad (2)$$

Without loss of generality, $N_1=N_2=N$ is assumed for the filter order throughout this paper. The Type-3 separable denominator transfer function of the 2-D filter can be rewritten in (3).

$$H(z_1, z_2) = \frac{Y(z_1, z_2)}{Y_3(z_1, z_2)} \cdot \frac{Y_3(z_1, z_2)}{X(z_1, z_2)} \quad (3)$$

where $Y(z_1, z_2) / Y_3(z_1, z_2)$ is defined in (4).

$$Y = Y_3 + \sum_{j=1}^N b_{0j} z_2^{-j} Y \quad (4)$$

where $Y=Y(z_1, z_2)$ and $Y_3=Y_3(z_1, z_2)$.

Therefore, $Y_3(z_1, z_2) / X(z_1, z_2)$ is generally expressed as

$$Y_3 = \sum_{i=0}^N \sum_{j=0}^N a_{ij} z_1^{-i} z_2^{-j} X + \sum_{i=1}^N b_{i0} z_1^{-i} Y_3 \quad (5)$$

where $X = X(z_1, z_2)$.

III. PROPOSED TYPE-3 2-D QUADRANTAL SYMMETRY FILTER ARCHITECTURE AND THE ERROR ANALYSIS

A 2-D magnitude response possesses quadrantal symmetry if $|H(z_1, z_2)| = |H(z_1^{-1}, z_2)|$ with $z_1 = e^{j\theta_1}$ and $z_2 = e^{j\theta_2}$, $\forall (\theta_1, \theta_2)$. Assuming the separable denominator transfer function in (2) is adopted, it will have quadrantal symmetry if $a_{ij} = a_{(N-i)j}$ and $b_{k0} = b_{0k}$ for all i, j, k . Applying the constraints to the transfer function in (2), equation (5) can be written as (6).

$$Y_3 = \sum_{j=1}^N b_{0j} z_1^{-j} Y_3 + v \cdot \sum_{j=0}^N a_{uj} (z_1^{-u} z_2^{-j}) X + \sum_{i=0}^{u-v} \sum_{j=0}^N a_{ij} (z_1^{-i} z_2^{-j} + z_1^{-(N-i)} z_2^{-j}) X \quad (6)$$

where $u = \lfloor N/2 \rfloor$ and $v = (N+1) \bmod 2$. Note that $\lfloor \cdot \rfloor$ denotes the largest integer that is smaller than or equal to \cdot . Considering (4) and (6) with $N=3$, a 3x3 2-D quadrantal symmetry IIR filter structure can be proposed in Fig. 1. Meanwhile, a new quadrantal symmetry FIR filter architecture can be obtained in Fig. 2 by setting b_{ij} to zero except $b_{00}=1$. In [2, 3], the study of product quantization errors propagating through the filter in fixed-point implementation has been discussed. Applying the assumption and approach mentioned in [2], the noise sources are uncorrelated in the linear noise model and linear decomposition can be utilized. Similarly to [2], the error signal notations are indicated in the figures throughout this paper. In Fig. 1, it is noted that the noise source e_{a00} are dispatched to path(s) with shared numerator a_{00} and so as other noise sources. In this paper the noise source and the noise source with delay are simplified to be regarded as independent ones. Thus, the signal notations e_1 and e_2 denote the linear errors as (7) and (8), respectively

$$e_1 = \sum_{j=1}^N e_{b0j} \quad (7)$$

$$e_2 = \sum_{j=1}^N e_{b0j} + v \cdot \sum_{j=0}^N e_{auj} + 2 \cdot \sum_{i=0}^{u-v} \sum_{j=0}^N e_{aij} \quad (8)$$

According to the linear decomposition of the linear noise model structure [2] in Fig. 1, since e_1 error/noise source passes through b_{0j} in block 1 and e_2 passes through b_{0i} in block 2 and b_{0j} in block 1 in Fig. 1, total variance of quantization error can be derived as

$$\sigma_{\text{Type3_Qua_IIR}}^2 = N \sigma_e^2 \sum_{n=-\infty}^{\infty} |h_{b2}[n]|^2 + [N + (N+1)^2] \sigma_e^2 \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |h_{b12}[m, n]|^2 \quad (9)$$

, where $\sigma_e^2 = 2^{-2B} / 12$, B is the fractional bit width after quantization, and $h_{b2}[n]$ and $h_{b12}[m, n]$ are defined in (10) and (11), respectively.

$$h_{b2}[n] \leftarrow \frac{z}{z-1} \rightarrow \frac{1}{1 - \sum_{j=0}^N b_{0j} z_2^{-j}} \quad (10)$$

$$h_{b12}[m, n] \leftarrow \frac{z}{z-1} \rightarrow \frac{1}{\left(1 - \sum_{i=0}^N b_{0i} z_1^{-i}\right) \cdot \left(1 - \sum_{j=0}^N b_{0j} z_2^{-j}\right)} \quad (11)$$

Similarly, the signal notation e_3 in Fig. 2 denote the linear errors as (12).

$$e_3 = v \cdot \sum_{j=0}^N e_{auj} + 2 \cdot \sum_{i=0}^{u-v} \sum_{j=0}^N e_{aij} \quad (12)$$

In the same way, the error analysis for Type-3 quadrantal-symmetry FIR filter in Fig. 2 is derived as follows.

$$\sigma_{\text{Type3_Qua_FIR}}^2 = \lfloor (N+1)^2 \rfloor \sigma_e^2 \quad (13)$$

IV. PROPOSED TYPE-3 2-D OCTAGONAL SYMMETRY FILTER ARCHITECTURE AND THE ERROR ANALYSIS

Octagonal symmetry is a combination of diagonal, four-fold rotational and quadrantal symmetries, where any two of the three symmetries are sufficient to create octagonal symmetry [12, 16]. According to the separable denominator transfer function in (2), it will have octagonal symmetry property if matching the following constraints on the coefficients: $a_{ij} = a_{ji} = a_{(N-i)j}$ and $b_{k0} = b_{0k}$ for all i, j, k . Utilizing these constraints on the transfer function in (2), equation (5) can be rewritten as (14).

$$Y_3 = \sum_{j=1}^N b_{0j} z_1^{-j} Y_3 + v a_{uu} z_1^{-u} z_2^{-u} X + v \cdot \sum_{i=0}^{u-v} a_{iu} (z_1^{-i} z_2^{-u} + z_1^{-u} z_2^{-(N-i)} + z_1^{-(N-i)} z_2^{-(N-u)} + z_1^{-(N-u)} z_2^{-i}) X + \sum_{i=0}^{u-v} a_{ii} (z_1^{-i} z_2^{-i} + z_1^{-i} z_2^{-(N-i)} + z_1^{-(N-i)} z_2^{-(N-i)} + z_1^{-(N-i)} z_2^{-i}) X + \sum_{i=0}^{u-v-1} \sum_{j=i+1}^{u-v} a_{ij} (z_1^{-i} z_2^{-j} + z_1^{-i} z_2^{-(N-j)} + z_1^{-(N-i)} z_2^{-j} + z_1^{-(N-i)} z_2^{-(N-j)}) X + z_1^{-j} z_2^{-i} + z_1^{-j} z_2^{-(N-i)} + z_1^{-(N-j)} z_2^{-i} + z_1^{-(N-j)} z_2^{-(N-i)}) X \quad (14)$$

By using (4) and (14) with $N=3$, the Type-3 octagonal symmetry IIR filter structure is shown in Fig. 3, where the e_1 noise source is the same as in (7) and e_4 is rewritten as (15).

$$e_4 = \sum_{j=1}^N e_{b0j} + v e_{auu} + 4v \cdot \sum_{i=0}^{u-v} e_{aii} + 4 \cdot \sum_{i=0}^{u-v} e_{aii} + 8 \cdot \sum_{i=0}^{u-v-1} \sum_{j=i+1}^{u-v} e_{aij} \quad (15)$$

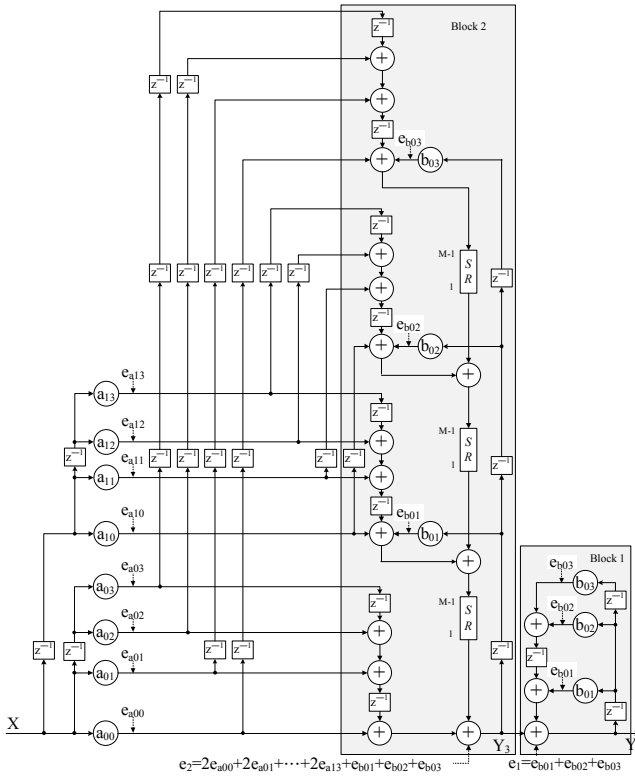


Fig. 1. Proposed Type-3 2-D quadrantal symmetry IIR filter structure with separable denominator for $N=3$.

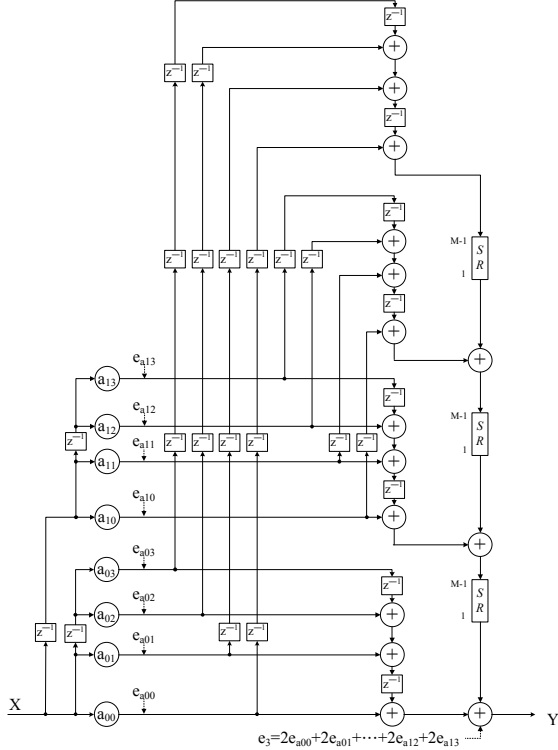


Fig. 2. Proposed Type-3 2-D quadrantal symmetry FIR filter structure for $N=3$.

According the linear decomposition of the linear noise model structure [2] in Fig. 3, since e_l error/noise sources passes through b_{oj} at the right hand side of the filter structure and e_4 passes through b_{oi} in the middle part and b_{oj} at the right hand side in Fig. 3, total variance of quantization error can be derived as

$$\sigma_{\text{Type3_Oct_IIR}}^2 = N\sigma_e^2 \sum_{n=-\infty}^{\infty} |h_{b2}[n]|^2 + [N + (N+1)^2] \sigma_e^2 \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |h_{b12}[m, n]|^2 \quad (16)$$

, where $h_{b2}[n]$ and $h_{b12}[m, n]$ are defined in (10) and (11), respectively. Similarly, we can also obtain a new octagonal symmetry FIR filter architecture in Fig. 4 by setting b_{ij} to zero except $b_{00}=1$, so as the noise source can be derived in (17)

$$e_5 = v e_{auu} + 4v \cdot \sum_{i=0}^{u-v} e_{aiu} + 4 \cdot \sum_{i=0}^{u-v} e_{aii} + 8 \cdot \sum_{i=0}^{u-v-1} \sum_{j=i+1}^{u-v} e_{aij} \quad (17)$$

Thus, the error analysis for Type-3 octagonal-symmetry FIR filter in Fig. 4 is derived as follows.

$$\sigma_{\text{Type3_Oct_FIR}}^2 = [(N+1)^2] \sigma_e^2 \quad (18)$$

According the error analysis of Type-3 filter structure with quadrantal and octagonal symmetry filters in (9) and (16), it implies that the Type-3 symmetry structures have similar error formulation. However, the above statement does not apply to all symmetry filter structures. For convenience of comparison, for example, the error analysis of Type-1 quadrantal symmetry and octagonal symmetry filters are listed as follows.

$$\sigma_{\text{Type1_Qua_IIR}}^2 = N\sigma_e^2 \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |h[m, n]|^2 + [N + v(N+1) + (u-v+1)(N+1)] \sigma_e^2 \sum_{n=-\infty}^{\infty} |h_{b2}[n]|^2 \quad (19)$$

$$\sigma_{\text{Type1_Oct_IIR}}^2 = N\sigma_e^2 \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |h[m, n]|^2 + [N + v + v(u-v+1) + (u-v+1) + (u-v)(u-v+1)/2] \sigma_e^2 \sum_{n=-\infty}^{\infty} |h_{b2}[n]|^2 \quad (20)$$

V. CONCLUSION

Two new two-dimensional (2-D) IIR and two new 2-D FIR filter architectures using quadrantal and octagonal symmetries are proposed. Besides, four error analysis derivations and expressions for the corresponding architectures have been discussed. According the error equation mentioned above, users can obtain the error values of filters and decide the bit width needed in the filter implementation.

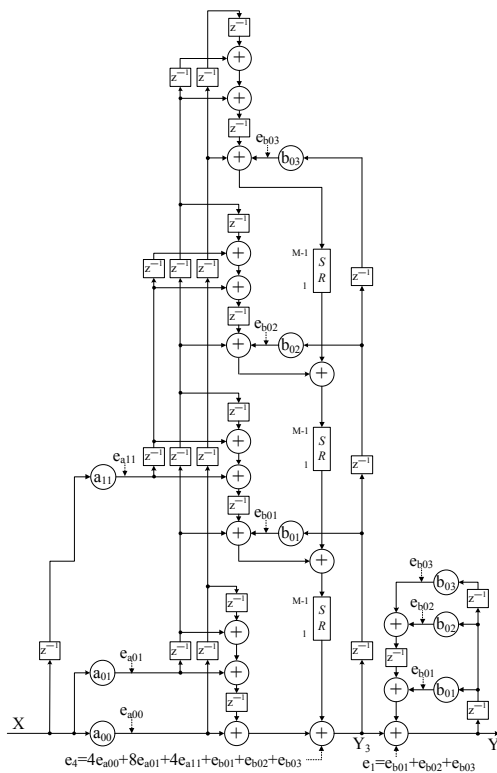


Fig. 3. Proposed Type-3 2-D octagonal symmetry IIR filter structure with separable denominator for $N=3$.

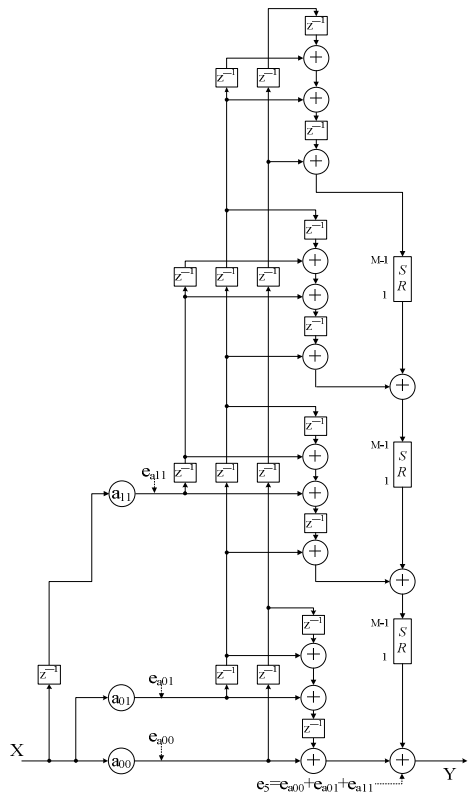


Fig. 4. Proposed Type-3 2-D octagonal symmetry FIR filter structure for $N=3$.

ACKNOWLEDGEMENT

The authors would like to thanks the supports of MOST 103-2221-E-009 -099 -MY3.

REFERENCES

- [1] D. E. Dudgeon and R. M. Mersereau, *Multidimensional Digital Signal Processing*. Englewood Cliffs, NJ: Prentice-Hall, 1984.
- [2] A. V. Oppenheim and R. W. Schaffer, *Discrete-Time Signal Processing*, Englewood Cliffs, NJ: Prentice-Hall, 1989, chapter 6.
- [3] Wu-Sheng Lu and Andreas Antoniou, *Two-Dimensional Digital Filters*. Marcel Dekker, Inc. New York, NY, USA 1992, chapter 11.
- [4] M. A. Sid-Ahmed, *Image Processing: Theory, Algorithms, and Architectures*. NY: McGraw-Hill, 1995.
- [5] M. Petrou and P. Bosdogianni, *Image Processing*, NY: Wiley, 2000.
- [6] R. M. Joshi, A. Madanayake, J. Adikari, and L. T. Bruton, "Synthesis and array processor realization of a 2-D IIR beam filter for wireless applications ", *IEEE Trans. Very Large Scale Integration (VLSI) Systems*, vol. 20, no. 12, pp. 2241 - 2254, Dec. 2012.
- [7] A. Madanayake, C. Wijenayake, D.G. Dansereau, T.K. Gunaratne, L.T. Bruton, and S.B. Williams, "Multidimensional (MD) circuits and systems for emerging applications including cognitive radio, radio astronomy, robot vision and imaging", *IEEE Circuits and Systems Magazine*, vol. 13, no. 1, pp. 10-43, 2013.
- [8] M. A. Sid-Ahmed, "A systolic realization for 2-D digital filters," *IEEE Trans. Acoust., Speech, Signal Processing*, vol. ASSP-37, pp. 560-565, Apr. 1989.
- [9] N. R. Shanbhag, "An improved systolic architecture for 2-D digital filters," *IEEE Trans. Signal Processing*, vol. 39, no. 5, pp. 1195-1202, May 1991.
- [10] L. D. Van, "A new 2-D systolic digital filter architecture without global broadcast," *IEEE Trans. VLSI Syst.*, vol. 10, no. 4, pp. 477-486, Aug. 2002
- [11] M. N. S. Swamy and P. K. Rajan, "Symmetry in 2-D filters and its application," in *Multidimensional Systems: Techniques and Applications* (S.G. Tzafestas, Ed.). New York: Marcel Dekkar, 1986, Ch.9.
- [12] H. C. Reddy, I. H. Khoo and P. K. Rajan, "2-D symmetry: theory and filter design applications," *IEEE Circuits and Systems Magazine*, vol. 3, pp. 4-33, 2003.
- [13] P. Y. Chen, L. D. Van, H. C. Reddy and C. T. Lin, "A new VLSI 2-D diagonal-symmetry filter architecture design," in *Proc. IEEE APCCAS*, Macao, China, pp. 320-323, Nov. 2008.
- [14] P. Y. Chen, L. D. Van, H. C. Reddy and C. T. Lin, "A new VLSI 2-D fourfold-rotational-symmetry filter architecture design," in *Proc. IEEE ISCAS*, Taiwan, pp. 93-96, May, 2009.
- [15] I. H. Khoo, H. C. Reddy, L. D. Van, and C. T. Lin, "General formulation of shift and delta operator based 2-D VLSI filter structures without global broadcast and incorporation of the symmetry," *Multidimensional Systems and Signal Processing*, 25, pp. 795-828, 2014.
- [16] P. Y. Chen, L. D. Van, I. H. Khoo, H. C. Reddy and C. T. Lin, "Power-Efficient and Cost-Effective 2-D Symmetry Filter Architectures," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol. 58, no. 1, pp. 112-125, Jan. 2011.
- [17] P. Y. Chen, L. D. Van, H. C. Reddy, and I. H. Khoo, "Area-efficient 2-D digital filter architectures possessing diagonal and four-fold rotational symmetries," in *Proc. International Conference on Information, Communications and Signal Processing (ICICS)*, Taiwan, Dec., 2013.