Exercise 7.4

10. Solution: $\mathscr{L}\{y\} = \dfrac{2}{((s-1)^2+1)^2}$, $y = e^t \sin t - te^t \cos t$.

17. Hint: The differential equation will be transformed into another differential equation in the Laplace domain. You can solve the differential equation in the Laplace domain by the first-order differential equation solution techniques. Then, you can transform the solution back to the time domain.

    Solution: $y(t) = 2t^3/3 + c_1 t^2$.

34. Solution: $\dfrac{3s+1}{s^2(s+1)^3}$.

44. Solution: $f(t) = 4e^{-t} - 7te^{-t} + 4t^2e^{-t}$.

Exercise 7.5

4. Solution: $y(t) = (1/4)\sin 4t \cdot u(t-2\pi)$.

10. Solution: $y = (t-1)e^{-(t-1)}u(t-1)$.

17. Solution: Disagree. Hint: check the initial conditions. Although the two equations are the same in the Laplace domain, one of the D.E. has no solution to the IVP.

Exercise 8.2

2. Solution: $\mathbf{X}(t) = c_1\begin{pmatrix}-2\\1\end{pmatrix}e^t + c_2\begin{pmatrix}1\\1\end{pmatrix}e^{4t}$.

24. Solution: $\mathbf{X}(t) = c_1\begin{pmatrix}3\\2\end{pmatrix}e^{6t} + c_2\left[\begin{pmatrix}3\\2\end{pmatrix}te^{6t} + \begin{pmatrix}1/2\\0\end{pmatrix}e^{6t}\right]$.

32. Solution: $\mathbf{X}(t) = 2\cdot\begin{pmatrix}-1\\0\\1\end{pmatrix}e^{-t} + 3\cdot\begin{pmatrix}1\\0\\1\end{pmatrix}e^t + 2\cdot\begin{pmatrix}0\\1\\0\end{pmatrix}e^t$.

Exercise 8.3

5. Solution: $\mathbf{X}(t) = c_1\begin{pmatrix}1\\-3\end{pmatrix}e^{3t} + c_2\begin{pmatrix}1\\9\end{pmatrix}e^{7t} + \begin{pmatrix}55/36\\-19/4\end{pmatrix}e^{t}$.

25. Solution: $\mathbf{X}(t) = c_1\begin{pmatrix}-\sin t\\ \cos t\end{pmatrix}e^{t} + c_2\begin{pmatrix}\cos t\\ \sin t\end{pmatrix}e^{t} + \begin{pmatrix}\cos t\\ \sin t\end{pmatrix}te^{t}$.

34. Solution: $\mathbf{X}(t) = \begin{pmatrix}3\\3\end{pmatrix}t - \begin{pmatrix}1\\4\end{pmatrix} + \begin{pmatrix}1\\1\end{pmatrix}\ln t$.

Exercise 8.4

10. Hint: $e^{-\mathbf{A}t} = \begin{pmatrix}e^{-t} & 0\\ 0 & e^{-2t}\end{pmatrix}$. Solution: $\mathbf{X}(t) = c_1\begin{pmatrix}1\\0\end{pmatrix}e^{t} + c_2\begin{pmatrix}0\\1\end{pmatrix}e^{2t} + \begin{pmatrix}-t-1\\ \frac{1}{2}e^{4t}\end{pmatrix}$.

16. Hint: $e^{-\mathbf{A}t} = \begin{pmatrix}2e^{3t}-e^{2t} & -2e^{3t}+2e^{2t}\\ e^{3t}-e^{2t} & -e^{3t}+2e^{2t}\end{pmatrix}$. Solution: $\mathbf{X}(t) = c_1\begin{pmatrix}2\\1\end{pmatrix}e^{3t} + c_2\begin{pmatrix}1\\1\end{pmatrix}e^{2t}$.